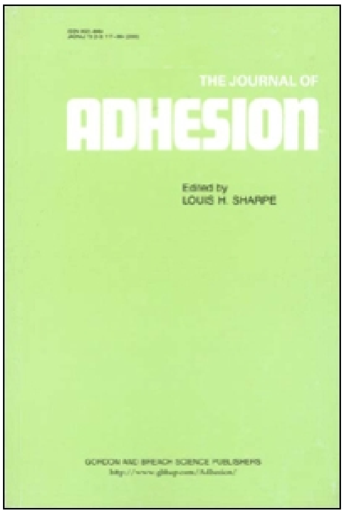

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Adhesive Fracture of Bonded Plate or Membrane Strips†

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One of the few exact non-linear solutions applying to adhesive debonding is for the deformations in a pressurized thin plate strip of infinite length subjected over its span to externally applied pressure and temperature. For example, if the ends of the span are clamped (bonded) to a substratum leaving an unbonded span to be loaded by pressure or temperature, critical values of these latter quantities, at which debonding may occur, can be calculated using an energy balance criterion. To date, only the limiting cases of the general thin plate solution have been used to deduce these critical pressure or temperature loadings: (1) the plate thickness to span ratio is sufficiently large that nearly all the strain energy is in bending, i.e., the "thick plate" case, and (2) the opposite case wherein the ratio is sufficiently small that mainly stretching energy is involved, i.e., the "membrane". This last case, for example, has applications to the adhesion of paints and coatings. The purpose of this paper is to present the results of calculations for pressure criticality over the range of all plate thickness between the aforementioned two previously available limiting cases.

TECHNICAL DISCUSSION

In order to assess the quality of an adhesive bond in an arbitrary geometrical configuration, it is necessary, from the standpoint of continuum mechanics at least, to measure the specific adhesive fracture energy (γ_a) in some known calibration geometry. One of the simplest illustrative geometries for this purpose is the cantilever beam bonded to a rigid substratum. The analysis follows that for the specific cohesive fracture energy (γ_c) for a split cantilever[1] (Figure 1). The strain energy (U) stored in the top elastic beam, assumed clamped at the end of the split, is one-half of the work done by the applied force (F) acting through the equilibrium displacement, namely

$$U = \frac{F}{2} \cdot \frac{FL^3}{3EI} \tag{1}$$

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in which $I = b(2t)^3/12$ is the moment of inertia, and E is the tensile Young's modulus. The incremental increase in new fracture energy (Γ), counting only that associated with the top beam, is $\partial\Gamma = \gamma_c \partial(Lb)$. Then equating this incremental work per unit area with the incremental change in strain energy $\partial U/\partial(Lb)$, one has

$$\gamma_c = \frac{\partial\Gamma}{\partial(Lb)} = \frac{\partial U}{\partial(Lb)} = \frac{F^2L^2}{2bEI} = \frac{6F^2L^2}{Eb^2(2t)^3} \tag{2}$$

which is the determining equation for γ_c providing that the force F is interpreted to be that at the instant of debond, F_{cr}. The similarity in analysis to an adhesively bonded top beam is immediately evident, because at the mid-plane, which is the locus of splitting, the deformation is zero in both cases. The only difference, therefore, is in the physical distinction between cohesive and adhesive separation, which is incorporated in this continuum mechanics analysis by distinguishing a γ_c from a γ_a. For the latter case one replaces γ_c in (2) by γ_a, the specific adhesive fracture energy, namely

$$\gamma_a = \frac{6F_{cr}^2L^2}{Eb^2(2t)^3} \tag{3}$$

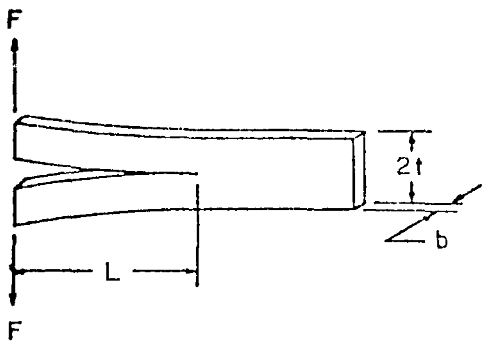

FIGURE 1 Split cantilever beam.

There are, however, other geometrical configurations which can be used as calibration cases to determine γ_a, such as the "blister" discussed in Ref. 2. Here it develops that for an initial blister debond radius of a and blister thickness h that

$$\gamma_a = \frac{3(1-\nu)^2}{32}\left[\frac{a}{h}\right]^3 \cdot \left[\frac{p_{cr}^2 a}{E}\right] \tag{4}$$

in which p_{cr} is the critical pressure under the round blister at the instant of fracture; ν is Poisson's ratio. In the earlier discussion[1] it was observed that this geometry represented certain physical situations occurring in "blistered" (as by heat) coatings. Usually, however, for thin coatings the thickness to radius ratio (h/a) is rather small thus more resembling a "membrane" than a "plate". Hence, an analysis based upon stretching energy, rather than the bending energy which predominates in (4), should be used. Unfortunately, however, this apparently simple problem is actually non-linear and has eluded analytical solution.[3]

In order to study the transition phenomenon as the plate thickness decreases from a case of nearly all bending energy ($h/a \gtrsim 0{\cdot}1$) to nearly all stretching energy ($h/a \to 0$), it is possible to utilize an existing analytical solution for a plate strip,[4, 5] including also the effect of temperature as well as pressure loading. The specific geometry to be considered consists of an infinitely long strip in the x-direction, of width, b, in the y-direction, and of thickness, h. While two cases were considered in the references, corresponding to simply supported and to clamped ends at $y = +b/2$, only the latter case will be analyzed here. Also the solution was presented for arbitrarily varying temperature through the plate thickness, but this adhesive debonding discussion will treat only the isothermal situation.

The exact limit isothermal solutions for the specific adhesive fracture energy γ_a were deduced[1] for all bending ("thick plate") and all stretching ("membrane") as:

$$\text{Thick plate: } \gamma_a = aE(p_{cr}/E)^2\left[\frac{2(1-\nu^2)}{3}\right]\cdot\left[\frac{a}{h}\right]^3 \tag{5}$$

$$\text{Membrane: } \gamma_a = aE(p_{cr}/E)^{\frac{4}{3}}\left[\frac{343(1-\nu^2)}{288}\cdot\frac{a}{h}\right]^{\frac{1}{3}} \tag{6}$$

The difference in functional behavior upon the critical pressure and geometry are to be noted.

The transition variation between these two limits is the object of this study. It utilized the basic analytical solution from Ref. 4, and the more general energy balance which follows from the second law of thermodynamics,[6-8] *i.e.*

$$\text{Work done on system} = \text{strain energy stored} + \text{new fracture surface energy created} \tag{7}$$

After some lengthy numerical computations, due to the complicated transcendental functions involved, the calculation of γ_a over the entire range of h/a was obtained. The results are shown in Figure 2 with (γ_a/Ea) as a parameter, along with the limit solutions (5) and (6). It may be noted that, as a function of γ_a/Ea, the thick plate solution governs at the larger values of h/a, while for the smaller thickness ratios the membrane is a close approximation. The transition range between the two is quite narrow.

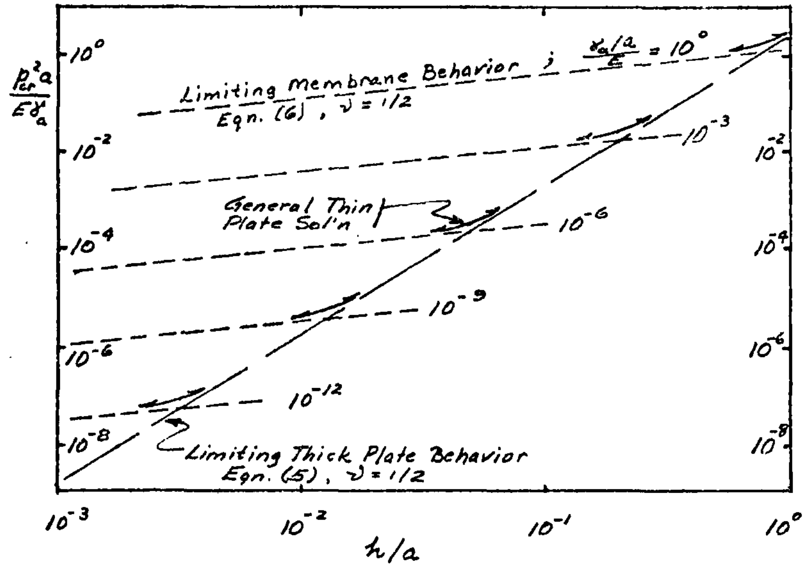

FIGURE 2 Critical pressure for debonding, showing narrowness of the transition region.

One is tempted to conclude that quantitatively, the same range of transition will exist for pressurized circular blisters. Also, should there be sufficient need, temperature variations could be introduced into the energy balance to simulate blistering of paint coatings due to either thermal buckling of the coating or tensile cracking in unbonded circular portions. Initial results for this thermal situation, however, may be sufficient (as taken from the limit cases of Ref. 1 for the plate strip) when used in conjunction with the shape of the transition region as presented for this one case.

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